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THOMSON REUTERS

[Singh *, 5(11): November 2018]
DOI- 10.5281/zenodo.1744947

ISSN 2348 – 8034
Impact Factor- 5.070

GLOBAL JOURNAL OF ENGINEERING SCIENCE AND RESEARCHES

STUDY OF PHOTON-STATISTICS IN INTENSITY DEPENDENT JAYNES-CUMMINGS MODEL

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ABSTRACT

Certain aspects of the field statistics in the intensity dependent Jaynes-Cummings Model has been investigated and the effect of detuning has been analyzed. We observe that the intensity dependence induces nonclassical photon states exhibiting antibunching.

Keywords: *Intensity dependent Jaynes-Cummings Model , Photon statistics, second order correlation function, Antibunching*

I. INTRODUCTION

In Quantum Optics, the interaction between a two-level atom and a quantized radiation field is a fundamental problem. The simplest model to deal with is the Rabi Model [1, 2] that describes the interaction of a two-level atom with a single mode of quantized electromagnetic field. Although widely studied over the past four decades, an exact analytical solution is still lacking and only numerical[3-5] and approximate analytical solutions are available[6, 7]. The most prevalent analytical approach to solve the Rabi model is known as rotating wave approximation (RWA) where the counter rotating terms are neglected. In this limit Rabi Hamiltonian is known as Jaynes-Cummings Hamiltonian and can be integrated exactly [8-10]. In spite of the simplicity of the JC model, The dynamics has turned out to be very rich and complex.

The Jaynes Cummings model (JCM) for the two-level atom strongly coupled to a single mode quantum field in a cavity allows one to investigate a series of qualitatively new physical effects in the evolution of the atomic state population. In particular the “collapse revival effect” (CRE) is of great interest as the pure quantum property of this system. Such nonclassical effects are of interest because they provide instances in which the quantum mechanical nature of the field manifests itself.

As the experimental techniques become even more sophisticated, it is now possible to investigate in laboratories new forms of light that have never been realized before. Three most striking examples are squeezed light, sub-Poissonian light and antibunched light[11-14]. Light with quantum fluctuations in one quadrature smaller than those associated with coherent light is said to be squeezed. Light whose photon number fluctuations are smaller than those of the Poisson distribution is called sub-Poisson or alternatively photon-number-squeezed light. Photon antibunching is a characteristic of light field with photons more equally spaced than a coherent laser field. It can also refer to sub-Poisson photon statistics, i.e. a photon number distribution for which variance is less than the mean.

In the studies of the theory of quantum optics, the antibunched light is considered significant because of its applications in optical communication and in interferometric techniques.

In this paper we investigate the photon statistics in the intensity dependent Jaynes Cummings models.

II. DESCRIPTION OF THE MODEL

In the Jaynes Cummings Model with intensity dependent coupling the coupling strength is assumed to depend on the number operator. This interaction is no longer linear in the field variables and represents an intensity dependent coupling. The intensity dependence of the Hamiltonian is practical in the case of cavity QED. This is due to the fact that the interaction takes place with a few photons. So, addition or destruction of a single photon changes the atom-field coupling constant substantially.

The Hamiltonian for this model is written as [10, 15,16]

$$\hat{H} = \frac{\hbar\omega_0}{2}\sigma_3 + \hbar\omega\,\hat{a}^\dagger\hat{a} + \hbar g(\hat{\sigma}_+\hat{R} + \hat{\sigma}_-\hat{R}^\dagger) \tag{1}$$

where $$\hat{R} = \hat{a}\sqrt{(\hat{N})}\ ,\quad \hat{R}^\dagger = \sqrt{(\hat{N})}\ \hat{a}^\dagger\ ,\qquad \hat{N} = \hat{a}^\dagger\,\hat{a} \tag{2}$$

For the Jaynes Cummings Model with Intensity dependent coupling the expression for the general state vector $|\psi(t)\rangle$ can be written as

$$|\psi(t)\rangle = \sum_{n=0}^{\infty}\ [a'_n(t)|a,n\rangle + b'_{n+1}(t)|b,n+1\rangle$$

(3)

The expression for the transition probability for this model is given by [10]

$$|b'_{n+1}(t)|^2 = \frac{4k^2(n+1)^2}{\{(\Delta\omega)^2 + 4k^2(n+1)^2\}}\sin^2[\frac{t}{2}\{(\Delta\omega)^2 + 4k^2(n+1)^2\}^{\frac{1}{2}}] \tag{4}$$

For the case of exact resonance the atom-field detuning $\Delta\omega = \omega - \omega_0 = 0$, so that we obtain

$$|b'_{n+1}(t)|^2 = \sin^2 gt(n+1)\quad ,\quad |a'_n(t)|^2 = 1 - \sin^2 gt(n+1) \tag{5}$$

If initially the field mode is prepared in a coherent state we have

$$|\psi(t)\rangle = \sum_{n=0}^{\infty} P_n(\bar{n})\,[a'_n(t)|a,n\rangle + b'_{n+1}(t)|b,n+1\rangle \tag{6}$$

and the expression for the population inversion becomes

$$W(t) = \sum_{n=0}^{\infty} P_n(\bar{n})\cos[2gt(n+1)] \tag{7}$$

where he Rabi frequency
$$\Omega(\bar{n}) = 2g(\bar{n}+1) \tag{8}$$

and

$$P_n(\bar{n}) = |\langle n|\alpha\rangle|^2 = |C_n(\alpha)|^2 = \exp(-\bar{n})\frac{\bar{n}^n}{n!} \tag{9}$$

$P_n(\bar{n})$ represents the coherent field probability distribution functions for photon numbers in Poisson statistics and $\bar{n}$ is the initial average number of photons.

ISSN 2348 – 8034
Impact Factor- 5.070

III. MEAN NUMBER OF PHOTONS IN THE MODES

Let us examine the dynamics of average photon number

The mean number of photons $\langle N(t)\rangle$ is obtained as

$$\langle N(t)\rangle = \langle a^{\dagger}(t)a(t)\rangle = \sum_{n=0}^{\infty} nP_n(n)|a_n'(t)|^2 + \sum_{n=0}^{\infty}(n+1)P_n(\bar{n})|b_{n+1}'(t)|^2 \tag{10}$$

Fig. (1) depicts the time variation of Mean Number of Photons for a) $\bar{n}=5$ **b)** $\bar{n}=10$ respectively.

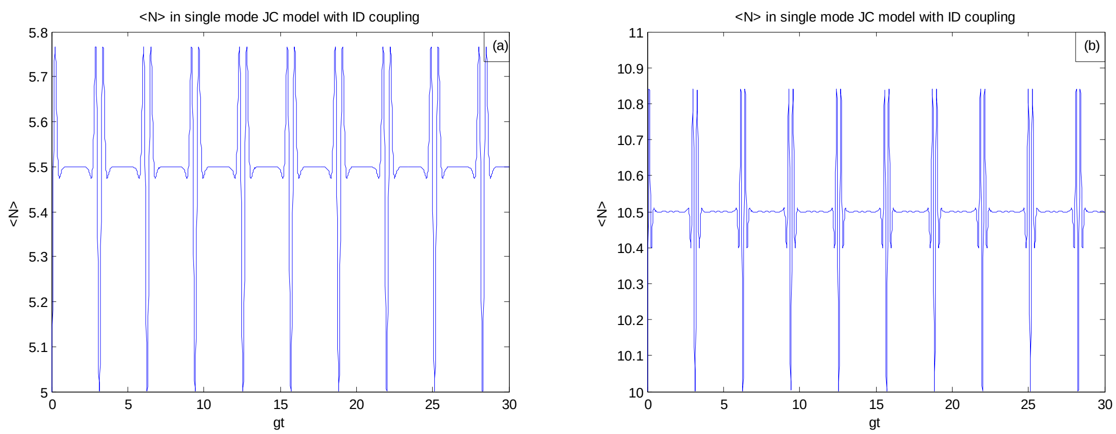

Fig.1.Time evolutions of the mean no of photons photon number for a) $\bar{n}=5$ ***b)*** $\bar{n}=10$

The most striking feature characterizing the quantum dynamics of the model with the intensity dependent interaction terms is the appearance of a strict periodicity in the collapses and revivals. It is evident from the plots that the collapses and revivals for the mean number of photons exhibit the same pattern of collapse and revival as for the atomic inversion[10]. . This is because the time behavior of the mean number of photons is determined by the same harmonic time functions with frequencies $\Omega(\bar{n})$ as that for atomic inversion. Since the Rabi frequency $\Omega(\bar{n})=2g(\bar{n}+1),$ becomes linear in quantum number so that an exact periodic evolution is observed in Fig. 1.

IV. FIELD STATISTICS

In this section we study the dynamics of the field statistics of our system, paying particular attention to the production of states of the field exhibiting nonclassical properties viz. possible production of antibunched light.

Second order correlation functions $G^{(2)}(t)$ can be represented as:

$$G^{(2)}(t) = 1 + \frac{\langle :(\Delta N(t))^2 :\rangle}{\langle N(t)\rangle^2}, \tag{11}$$

where $\langle N(t)\rangle$ is the mean number of photons in the mode already obtained in Eq. (10) and

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$$\langle : (\Delta N(t))^2 : \rangle = \langle a^\dagger(t) a^\dagger(t) a(t) a(t) \rangle - \langle N(t) \rangle^2 \tag{12}$$

is the normally ordered uncertainty in the number of photons. The light is nonclassical, displaying the sub-Poission statistics, whenever $G^{(2)}(t) < 1$ or equivalently whenever $\langle : (\Delta N(t))^2 : \rangle < 0$. Here we are actually calculating the zero time delay coherence function, hence the states satisfying these conditions are referred to as antibunched .

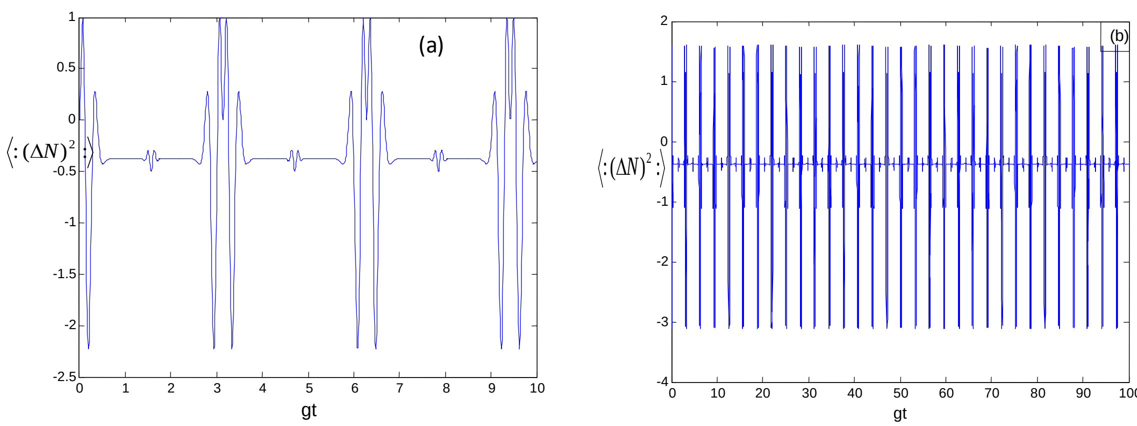
In order to calculate the normally ordered variance $\langle : (\Delta N(t))^2 : \rangle$, we need to calculate the quantity $\langle a^\dagger(t) a^\dagger(t) a(t) a(t) \rangle$.

We obtain

$$\langle a^\dagger(t) a^\dagger(t) a(t) a(t) \rangle = \sum_{n=0}^{\infty} n(n-1) P_n(\bar{n}) \left| a'_n(t) \right|^2 + \sum_{n=0}^{\infty} (n+1) n P_{n_1}(\bar{n}_1) \left| b'_{n+1}(t) \right|^2 \tag{13}$$

Using Eq. (13) and (10), $\langle : (\Delta N(t))^2 : \rangle$ defined through Eq. (12) is evaluated numerically.

In Fig.2 (a) and 2(b) we plot the time dependence of the quantity $\langle : (\Delta N(t))^2 : \rangle$ with the coherent initial state of the cavity field with $\bar{n} = 10$ and $\bar{n} = 20$ respectively. It is observed that the uncertainty in the number of photons is negative except at short time intervals. This indicates that the cavity field predominately exhibits a non-classical state with sub-Poisson statistics exhibiting the antibunching of photons.



ISSN 2348 – 8034
Impact Factor- 5.070

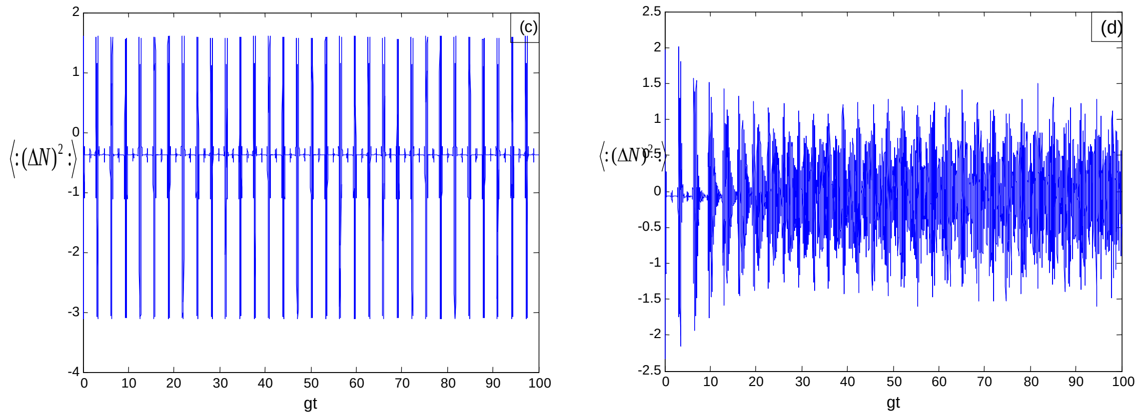


Fig.2. Time evolutions of $\langle :(\Delta N(t))^2 : \rangle$ ***for finite detunings: a)*** $\bar{n} = 10$, $\Delta\omega = 0$ ***b)*** $\bar{n} = 20, \Delta\omega = 0$ ***c)*** $\bar{n} = 20, \Delta\omega = 5g$ ***d)*** $\bar{n} = 20, \Delta\omega = 10g$

V. RESULTS AND DISCUSSION

It is observed that in Fig. (2) that $\langle :(\Delta N(t))^2 : \rangle$ exhibits the same form of collapse and revivals as does the population difference W (t) [10].

An increase in the initial average number of photons increases the maximum and minimum values at short time intervals.

We also study the time dependence of $\langle :(\Delta N(t))^2 : \rangle$ when detuning exists. The time evolution of $\langle :(\Delta N(t))^2 : \rangle$ for $\bar{n} = 20$ with different value of the detuning $\Delta\omega$ are shown in in Fig.2(c) and 2(d) respectively.

It is observed that, with the increase in detuning there is small gradual decrease in the amplitude of oscillations but the periodicity of the revivals is maintained. Increase in detuning decreases the antibunching.
It is observed that the uncertainty in the normally ordered variance is always negative except at short time regular intervals. This indicates that in the cavity field mainly exhibits a nonclassical state with sub-Poisson statistics exhibiting the antibunching of photons.

VI. CONCLUSION

We have examined some aspects of photon statistics for the model and observe that the cavity field mainly exhibits a nonclassical state with sub-Poisson statistics exhibiting the antibunching of photons. Increase in detuning decreases the amplitude of oscillations and antibunching.

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ISSN 2348 – 8034
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